Capacitated Domination and Covering

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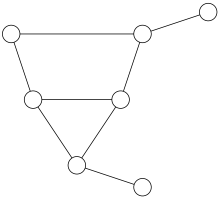
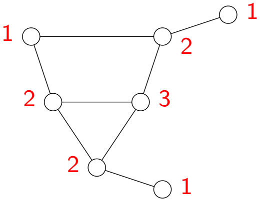
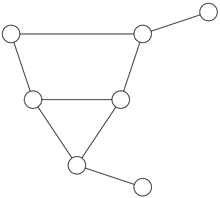
Capacitated Dominating Set and Capacitated Vertex Cover

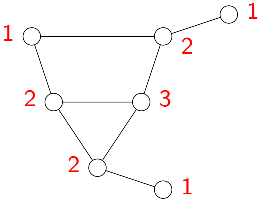
Dominating Set:





Vertex Cover:





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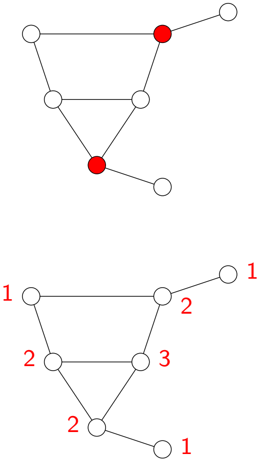

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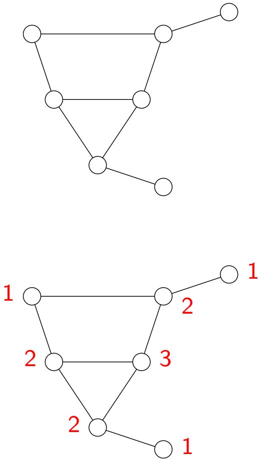

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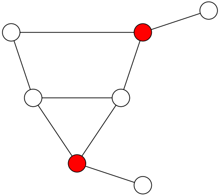

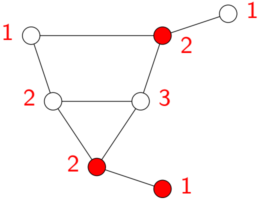

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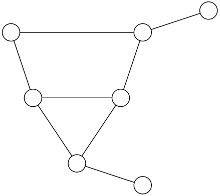
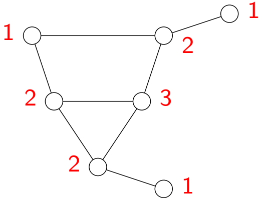
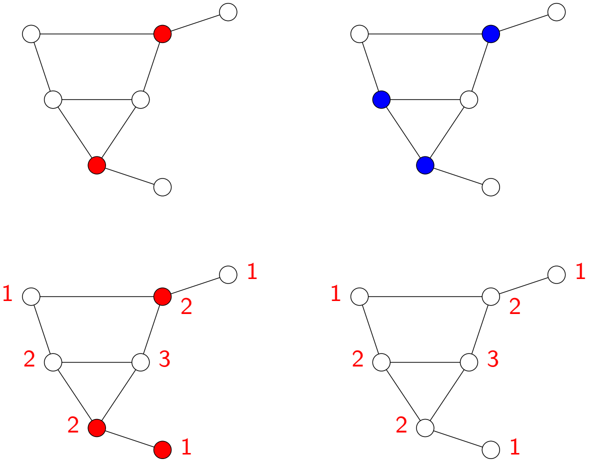




Capacity: Number of neighbors that a vertex can dominate.

Capacitated Dominating Set and Capacitated Vertex Cover

Dominating Set: Vertex Cover:



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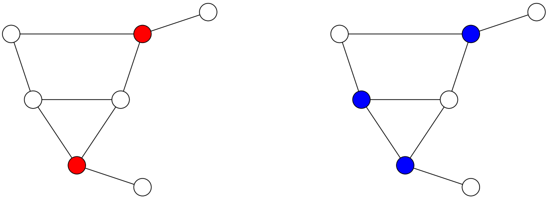
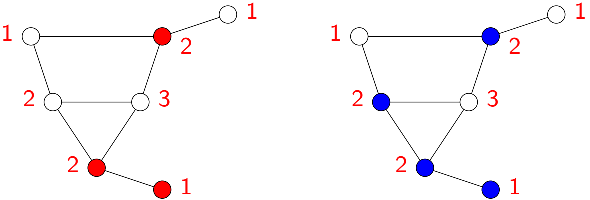




Capacity: Number of neighbors that a vertex can dominate.

Capacitated Dominating Set and Capacitated Vertex Cover

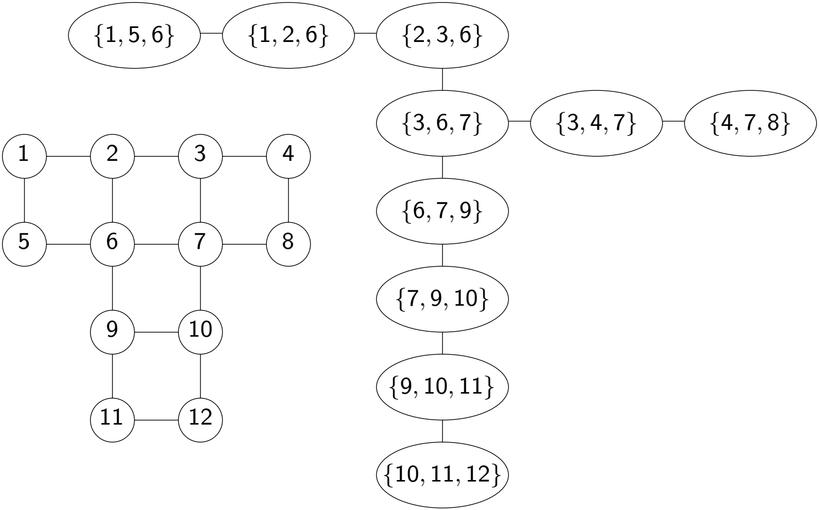
Input:
A graph $G = (V, E)$ with vertex capacities $\text{cap}(v)$, a positive integer k.

Question Is there a dominating set (a vertex cover) of size k that respects the given capacities?

Both problems are NP-complete.

Treewidth

Measures how “treelike” a graph is.



Fixed-Parameter Tractability

- FPT with respect to a parameter k
  $\Leftrightarrow$ running time $f(k) \cdot n^{O(1)}$.

- FPT with respect to a "combined" parameter k_1, k_2
  $\Leftrightarrow$ running time $f(k_2, k_2) \cdot n^{O(1)}$.

  Example: $O(k^{\mathrm{tw}} \cdot n)$

- W[1]-hardness: Basic concept for parameterized intractability.

Overview

	VC	CVC	DS	CDS
tw	FPT $O(2^{\text{tw}} \cdot n)$		FPT $O(4^{\text{tw}} \cdot n)$ [1]	
k	FPT $O(1.28^k + kn)$ [2]		W[2]-h.	
k, tw	FPT		FPT	

[1] [Alber et al., Algorithmica, 2002]
[2] [Chen et al., MFCS '06]
[3] [Guo et al., Theory Comput. Syst., 2007]

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Overview

	VC	CVC	DS	CDS
tw	FPT $O(2^{\text{tw}} \cdot n)$	W[1]-h.	FPT $O(4^{\text{tw}} \cdot n)$ [1]	W[1]-h.
k	FPT $O(1.28^k + kn)$ [2]	FPT $O(1.2^{k^2} + n^2)$ [3] $2^{O(k \log(k))} n^{O(1)}$	W[2]-h.	W[2]-h.
k, tw	FPT	FPT $2^{O(\text{tw} \log(k))} n^{O(1)}$	FPT	W[1]-h.

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Structure of the Talk

- Capacitated Dominating Set is W[1]-hard for the combined parameter k, tw
- Dynamic Programming for Capacitated Vertex Cover with parameter k, tw
- Capacitated Vertex Cover with parameter k

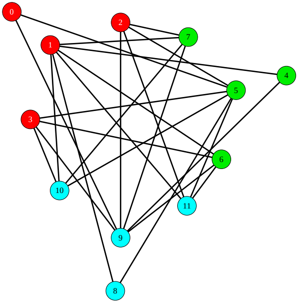
CDS is W[1]-hard w.r.t. the parameter k, tw

Parameterized reduction:

- Same basic idea as polynomial-time reduction.
- New parameter must depend only on the old parameter: $k' = f(k)$.
- (Reduction may cost $f(k) \cdot n^{O(1)}$ time.)

CDS is W[1]-hard w.r.t. the parameter k, tw

Reduction from Multicolored Clique.



$, 3$

Multicolored Clique

Input: A k-colored undirected graph and a positive integer k.

Question: Is there a (multicolored) clique of size k?

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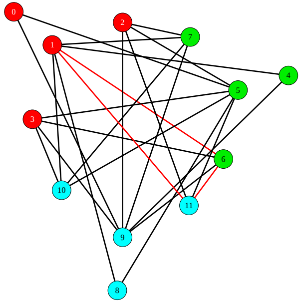

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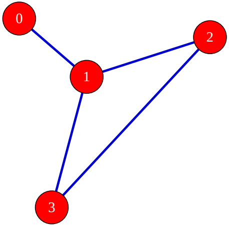

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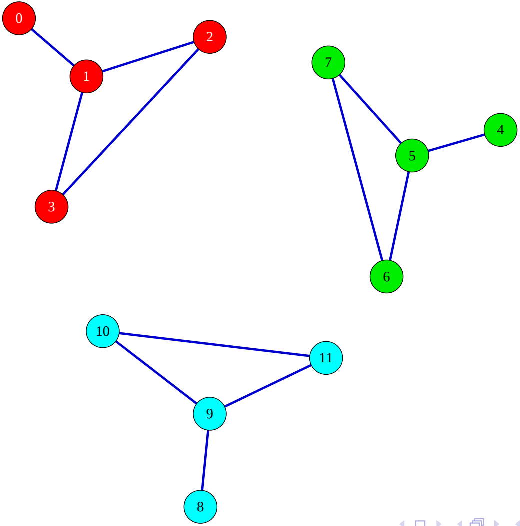

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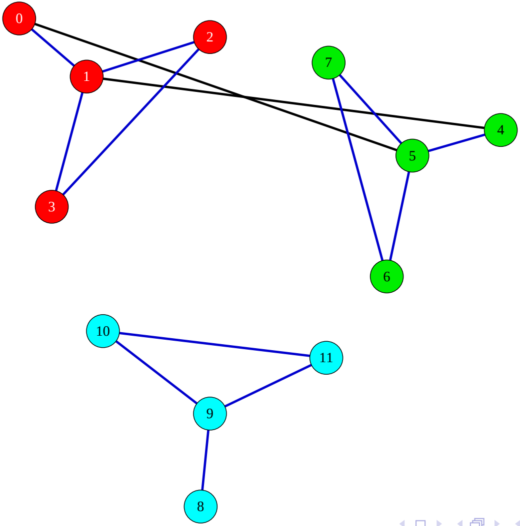

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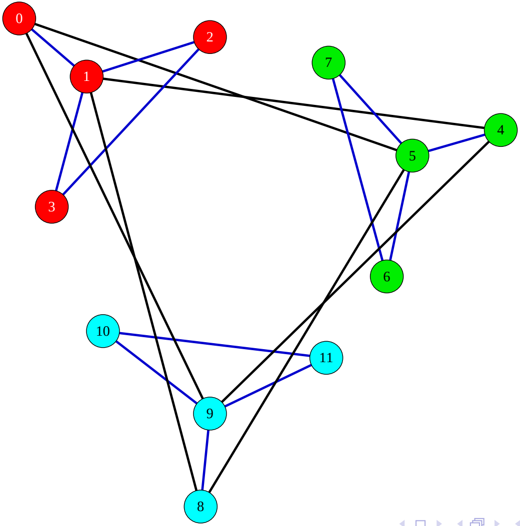

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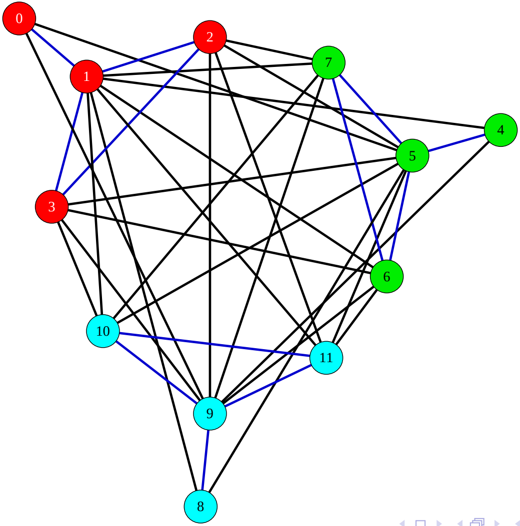

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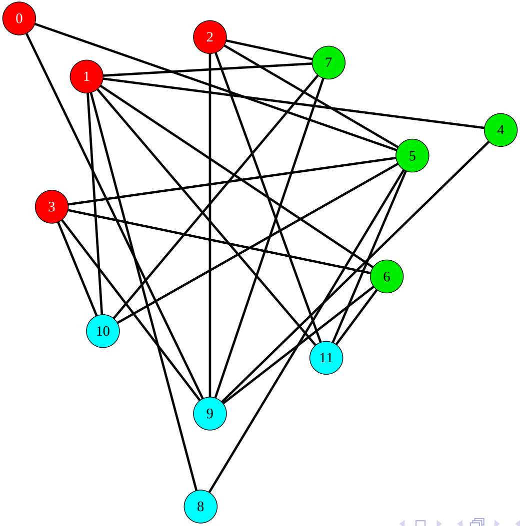

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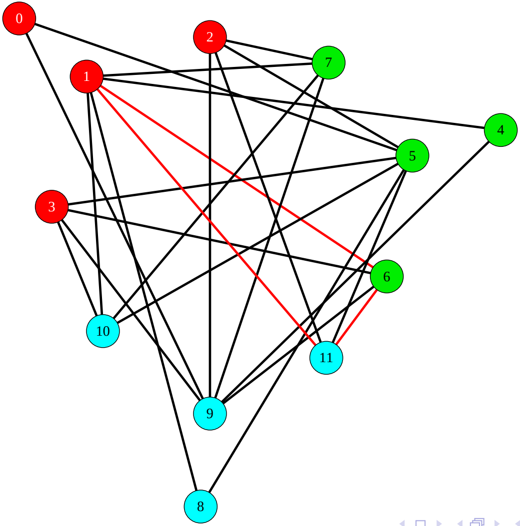

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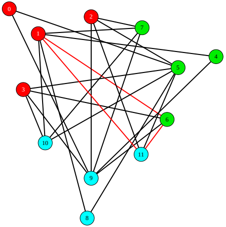
Reformulation of Multicolored Clique[4]:



$, 3$

Question: Is there a set E' of $\binom{k}{2}$ edges and a set V' of k vertices such that

- E' contains an edge of every "edge color",
- V' contains a vertex of every color, and
- $\{v, w\} \in E' \rightarrow v, w \in V'$?

[4] [Fellows et al., manuscript, 2008]

CDS is W[1]-hard w.r.t. the parameter k, tw

For an instance (G, k) of Multicolored Clique, construct an instance (G', cap, k') of Capacitated Dominating Set such that
– k' is bounded by some $f_1(k)$, and
– the treewidth of G' is bounded by some $f_2(k)$.

We reduce to Marked Capacitated Dominating Set.

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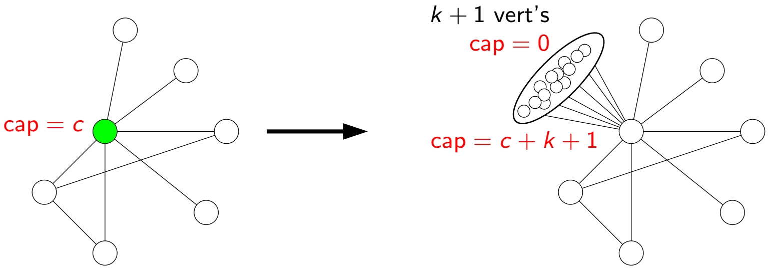

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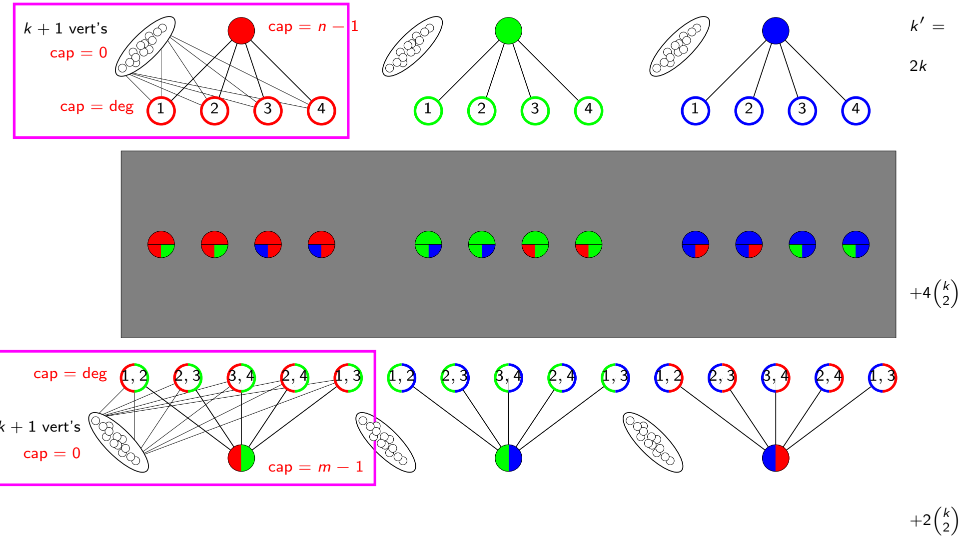

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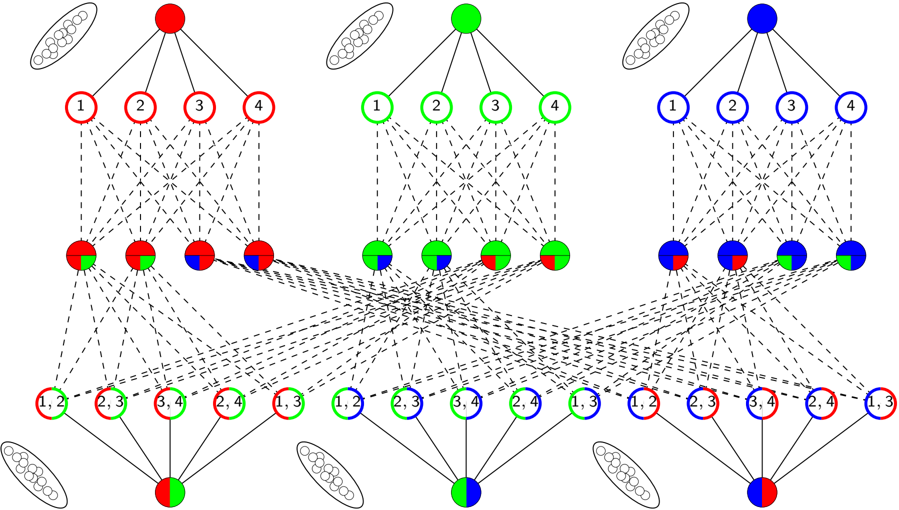

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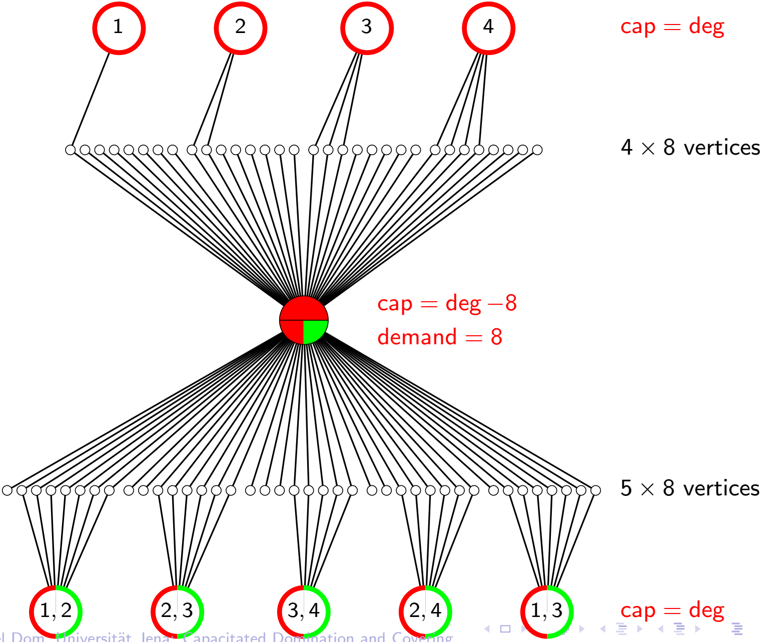

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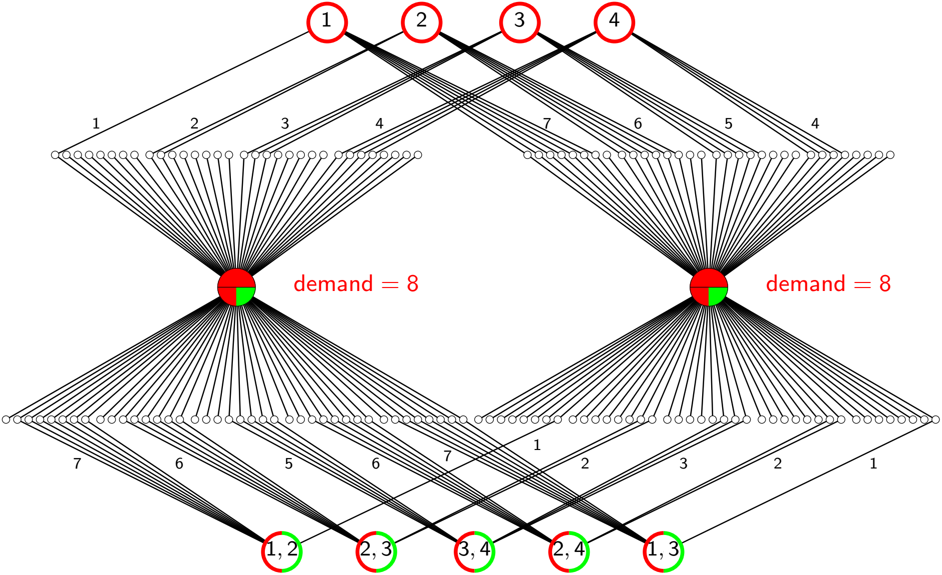

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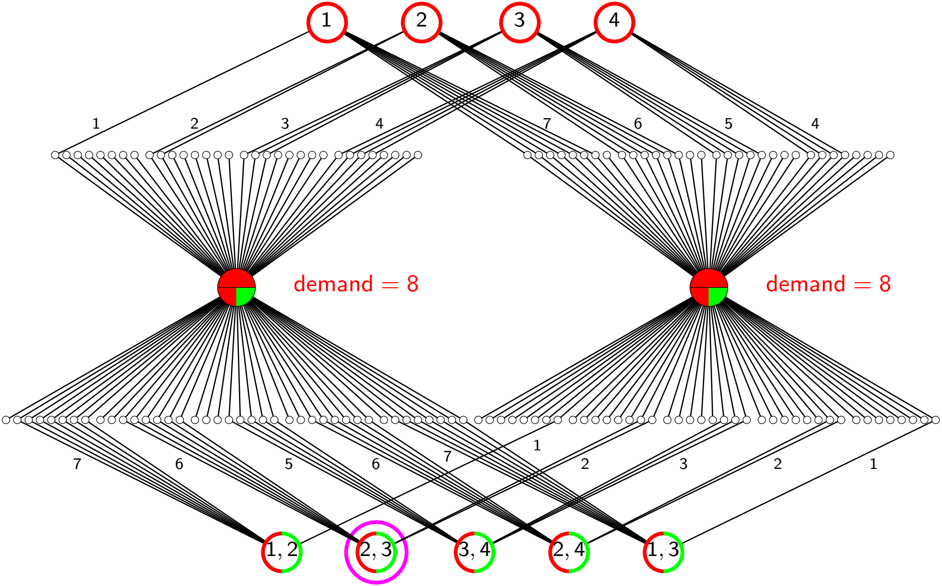

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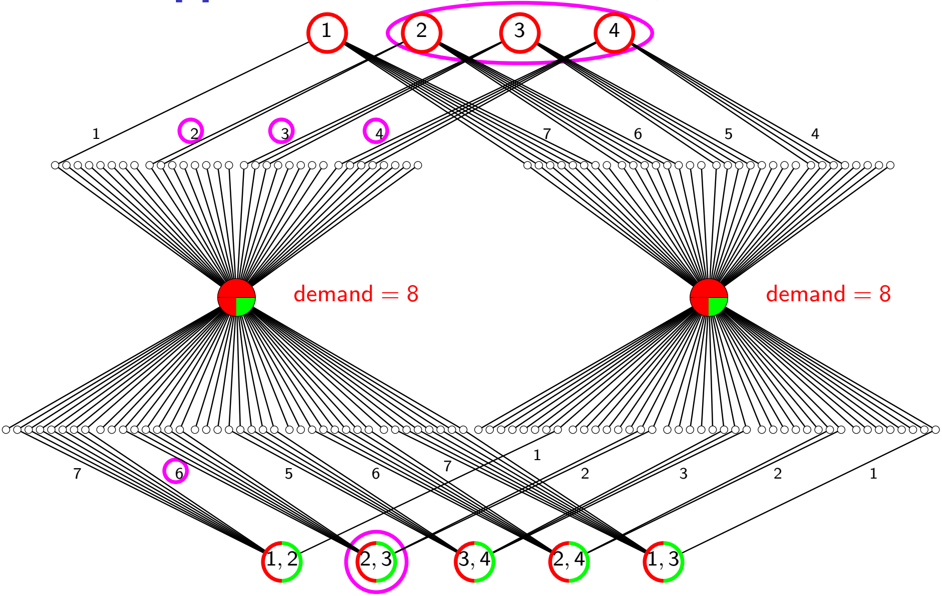

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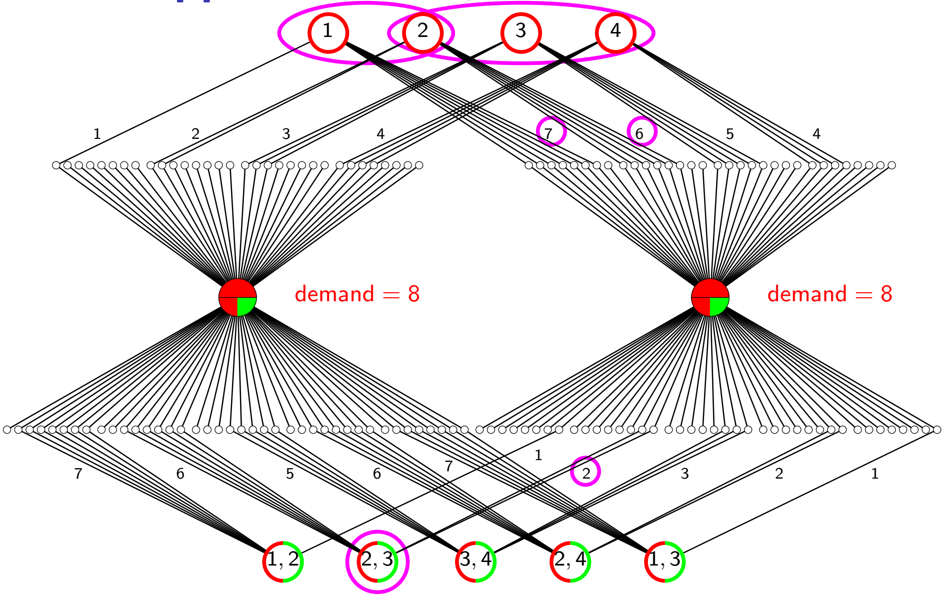

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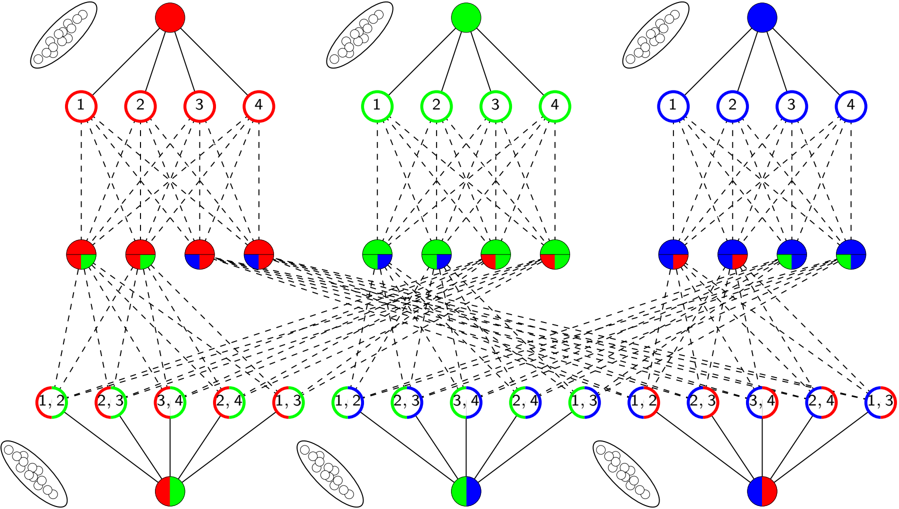

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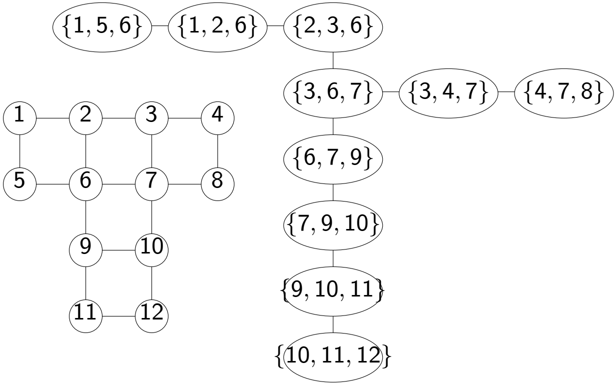
Dynamic Programming for Capacitated Vertex Cover with parameter k, tw



Main observation: At most k edges of every vertex v can be covered by neighbors of v.

$\Rightarrow$ $O(k^{\text{tw}})$ possibilities per bag.

$\Rightarrow$ Running time $2^{O(\text{tw} \log(k))} n^{O(1)}$.

Capacitated Vertex Cover with parameter k

1. Solve Vertex Cover in $O(1.28^k + kn)$ time[5].
2. Solution yields tree decomposition of width k.
3. Solve Capacitated Vertex Cover in $2^{O(\mathrm{tw}\log(k))} n^{O(1)}$ time.

$\Rightarrow$ Running time $2^{O(k\log(k))} n^{O(1)}$.

[5] [Chen et al., MFCS '06]

Open Questions

Is Capacitated Dominating Set FPT with respect to the parameter k in planar graphs?