Fixed-Parameter Algorithms for Consecutive Ones Submatrix Problems

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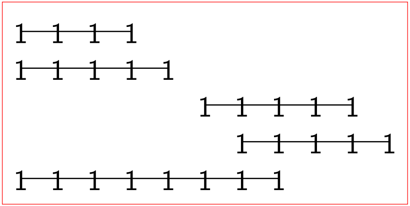
Consecutive Ones Property (C1P)



A 0/1-matrix has the C1P if its columns can be permuted such that in each row the ones form a block.

Consecutive Ones Property (C1P)

Example for a matrix having the C1P:

1	2	3	4	5
1	1			1
1		1		1
1		1	1	

Consecutive Ones Property (C1P)

Example for a matrix having the C1P:

1	2	3	4	5
1	1			1
1		1		1
1		1	1	

2	5	1	3	4
1	1	1		
	1	1	1	
		1	1	1

Consecutive Ones Property (C1P)

Examples for matrices **not** having the C1P:

1	1	0
0	1	1
1	0	1

1	1	0	0
0	1	1	0
0	0	1	1
1	0	0	1

1	1	0	0	0
0	1	1	0	0
0	0	1	1	0
0	0	0	1	1
1	0	0	0	1

1	1	0	0
0	1	1	0
0	1	0	1

1	1	0	0	0	0
0	0	1	1	0	0
0	0	0	0	1	1
1	0	1	0	1	0

Consecutive Ones Property (C1P)

The Consecutive Ones Property...

- ...expresses "locality" of the input data.
- ...appears in many applications, e.g.
  - in railway system optimization
    [Ruf, Schöbel, Discrete Optimization, 2004;
    Mecke, Wagner, ESA '04],
  - bioinformatics
    [Christof, Oswald, Reinelt, IPCO '98;
    Lu, Hsu, J. Comp. Biology, 2003].
- ...can be recognized in polynomial time
  [Booth, Lueker, J. Comput. System Sci., 1976;
  Meidanis, Porto, Telles, Discrete Appl. Math., 1998;
  Habib, McConnell, Paul, Viennot, Theor. Comput. Sci., 2000,
  Hsu, J. Algorithms, 2002; McConnell, SODA '04].
- ...is subject of current research
  [Hajiaghayi, Ganjali, Inf. Process. Lett., 2002;
  Tan, Zhang, Algorithmica, 2007].

Problem Definition

Min-COS-C (Min-COS-R)
Given: A matrix M and a positive integer k.
Question: Can we delete at most k columns (at most k rows) such that the resulting matrix has the C1P?

Min-COS-C is NP-complete even on $(2,3)$- and $(3,2)$-matrices
[Hajiaghayi, Ganjali, Inform. Process. Letters, 2002;
Tan, Zhang, Algorithmica, 2007].
Min-COS-R is NP-complete even on $(3,2)$-matrices
[Hajiaghayi, Ganjali, Inform. Process. Letters, 2002].

Problem Overview

(1's per col, 1's per row)	Max-COS-C	Min-COS-C
$(3, 2)$	0.5-approx[1]	
$(\infty, 2)$	• No const. approx.[1]	
(∞, Δ)	• No const. approx.[1]	
$(2, 3)$	0.8-approx[1]	
$(2, \infty)$	0.5-approx[1]	
(Δ, ∞)		

[1] [Tan, Zhang, Algorithmica, 2007]

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(1's per col, 1's per row)	Max-COS-C	Min-COS-C
$(3, 2)$	0.5-approx[1]	
$(\infty, 2)$	• No const. approx.[1] • W[1]-hard	• No 2,72-approx. • Problem kernel
(∞, Δ)	• No const. approx.[1] • W[1]-hard	• $(\Delta + 2)$-approx. • $O((\Delta+2)^k \cdot \Delta^{O(\Delta)} \cdot \|M\|^{O(1)})$-alg.
$(2, 3)$	0.8-approx[1]	
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(Δ, ∞)		

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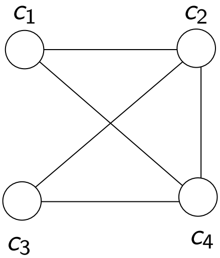
[1] [Tan, Zhang, Algorithmica, 2007]

Min-COS-C on $(\infty, 2)$-Matrices

Min-COS-C is equivalent to Induced Disjoint Paths Subgraph (IDPS).

Induced Disjoint Paths Subgraph (IDPS)
Given: A graph G and a positive integer k.
Question: Can we delete at most k vertices of G such that the resulting graph is a vertex-disjoint disjoint union of paths?

c_1	c_2	c_3	c_4
1	0	0	1
1	1	0	0
0	0	1	1
0	1	1	0
0	1	0	1



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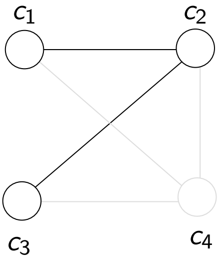


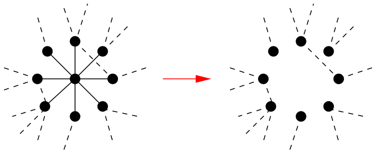
Problem Kernel for IDPS

Theorem: IDPS with parameter k admits a problem kernel with $O(k^2)$ vertices and $O(k^2)$ edges.

Data reduction rules:

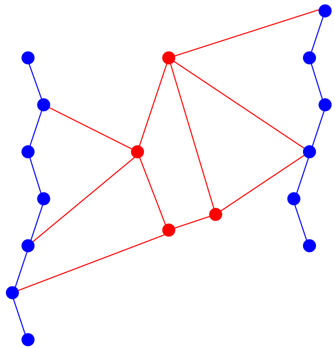
1. If a degree-two vertex v has two degree-at-most-two neighbors u, w with $\{u, w\} \notin E$, remove v from G and connect u, w by an edge.

2. If a vertex v has more than $k + 2$ neighbors, then remove v from G, add v to the solution, and decrease k by one.



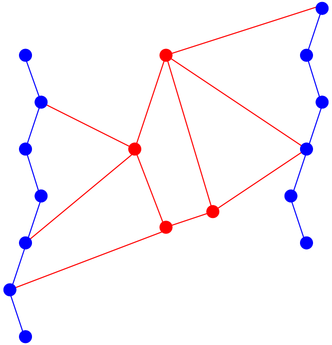
Problem Kernel for IDPS

- At most k red vertices.



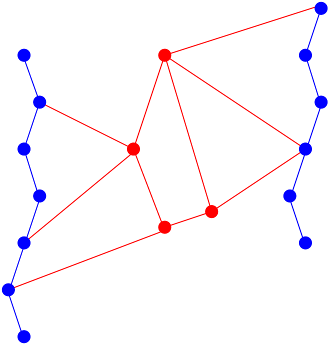
Problem Kernel for IDPS

- At most k red vertices.
- They have at most $k \cdot (k+2)$ blue neighbors.



Problem Kernel for IDPS

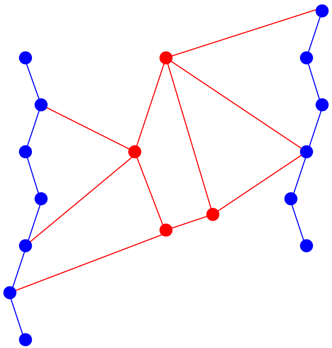
- At most k red vertices.
- They have at most $k \cdot (k + 2)$ blue neighbors.
- At least every third blue vertex must be a neighbor of a red vertex.



Problem Kernel for IDPS

- At most k red vertices.
- They have at most $k \cdot (k+2)$ blue neighbors.
- At least every third blue vertex must be a neighbor of a red vertex.

$\Rightarrow k + 3 \cdot k \cdot (k+2)$ vertices.
$\Rightarrow k \cdot (k+2) + 3 \cdot k \cdot (k+2) - 1$ edges.



Min-COS-C / Min-COS-R on (∞, Δ)-Matrices

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(Δ, ∞)		

[1] [Tan, Zhang, Algorithmica, 2007]

Min-COS-C / Min-COS-R on (∞, Δ)-Matrices

$$M_{\mathrm{I}_p} = \left.\overbrace{\begin{array}{ccccc} \mathbf{1} & \mathbf{1} & 0 & \cdots & 0 \\ 0 & \mathbf{1} & \mathbf{1} & 0\cdots & 0 \\ & & \cdots & & \\ 0 & \cdots & 0 & \mathbf{1} & \mathbf{1} \\ \hline \mathbf{1} & 0 & \cdots & 0 & \mathbf{1} \end{array}}^{p+2}\right\} p+2, \quad p \geq 1$$

$$M_{\mathrm{II}_p} = \left.\overbrace{\begin{array}{cccccc|c} \mathbf{1} & \mathbf{1} & 0 & & \cdots & & 0 \\ 0 & \mathbf{1} & \mathbf{1} & 0 & \cdots & & 0 \\ & & & \cdots & & & \\ 0 & \cdots & & 0 & \mathbf{1} & \mathbf{1} & 0 \\ \hline 0 & \mathbf{1} & & \cdots & & & \mathbf{1} \\ \mathbf{1} & & \cdots & & \mathbf{1} & 0 & \mathbf{1} \end{array}}^{p+3}\right\} p+3, \quad p \geq 1$$

$$M_{\mathrm{III}_p} = \left.\overbrace{\begin{array}{cccccc|c} \mathbf{1} & \mathbf{1} & 0 & & \cdots & & 0 \\ 0 & \mathbf{1} & \mathbf{1} & 0 & \cdots & & 0 \\ & & & \cdots & & & \\ 0 & \cdots & & 0 & \mathbf{1} & \mathbf{1} & 0 \\ \hline 0 & \mathbf{1} & & \cdots & \mathbf{1} & 0 & \mathbf{1} \end{array}}^{p+3}\right\} p+2, \quad p \geq 1$$

$$M_{\mathrm{IV}} = \begin{array}{|cccccc|} \hline \mathbf{1} & \mathbf{1} & 0 & 0 & 0 & 0 \\ 0 & 0 & \mathbf{1} & \mathbf{1} & 0 & 0 \\ 0 & 0 & 0 & 0 & \mathbf{1} & \mathbf{1} \\ \mathbf{1} & 0 & \mathbf{1} & 0 & \mathbf{1} & 0 \\ \hline \end{array} \qquad M_{\mathrm{V}} = \begin{array}{|ccccc|} \hline \mathbf{1} & \mathbf{1} & 0 & 0 & 0 \\ 0 & 0 & \mathbf{1} & \mathbf{1} & 0 \\ \mathbf{1} & \mathbf{1} & \mathbf{1} & \mathbf{1} & 0 \\ \mathbf{1} & 0 & \mathbf{1} & 0 & \mathbf{1} \\ \hline \end{array}$$

Theorem: A matrix has the C1P iff it contains none of the shown matrices.

[Tucker, Journal of Combinatorial Theory (B), 1972]

Min-COS-C / Min-COS-R on (∞, Δ)-Matrices

$$\overbrace{\left.\begin{array}{|ccccc|}\hline \mathbf{1} & \mathbf{1} & 0 & \cdots & 0 \\ 0 & \mathbf{1} & \mathbf{1} & 0\cdots & 0 \\ & & \ddots & & \\ 0 & \cdots & 0 & \mathbf{1} & \mathbf{1} \\ \hline \mathbf{1} & 0 & \cdots & 0 & \mathbf{1} \\ \hline \end{array}\right\}}^{p+2} p+2 \qquad M_{\mathrm{I}_p},\ p \geq 1$$

$$\overbrace{\left.\begin{array}{|ccccc|c|}\hline \mathbf{1} & \mathbf{1} & 0 & & \cdots & 0 \\ 0 & \mathbf{1} & \mathbf{1} & 0 & \cdots & 0 \\ & & & \ddots & & \\ 0 & \cdots & 0 & \mathbf{1} & \mathbf{1} & 0 \\ \hline 0 & \mathbf{1} & & \cdots & & \mathbf{1} \\ \mathbf{1} & & \cdots & \mathbf{1} & 0 & \mathbf{1} \\ \hline \end{array}\right\}}^{p+3} p+3 \qquad M_{\mathrm{II}_p},\ p \geq 1$$

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$$\begin{array}{|cccccc|}\hline \mathbf{1} & \mathbf{1} & 0 & 0 & 0 & 0 \\ 0 & 0 & \mathbf{1} & \mathbf{1} & 0 & 0 \\ 0 & 0 & 0 & 0 & \mathbf{1} & \mathbf{1} \\ \mathbf{1} & 0 & \mathbf{1} & 0 & \mathbf{1} & 0 \\ \hline \end{array} \qquad M_{\mathrm{IV}}$$

$$\begin{array}{|ccccc|}\hline \mathbf{1} & \mathbf{1} & 0 & 0 & 0 \\ 0 & 0 & \mathbf{1} & \mathbf{1} & 0 \\ \mathbf{1} & \mathbf{1} & \mathbf{1} & \mathbf{1} & 0 \\ \mathbf{1} & 0 & \mathbf{1} & 0 & \mathbf{1} \\ \hline \end{array} \qquad M_{\mathrm{V}}$$

Approach: Use a search tree algorithm.

Repeat:

1. Search for a "forbidden submatrix".
2. Branch on which of its columns has to be deleted.

Min-COS-C / Min-COS-R on (∞, Δ)-Matrices

$$\underbrace{\overbrace{\begin{array}{|cccccc|}\hline \mathbf{1} & \mathbf{1} & 0 & \cdots & & 0 \\ 0 & \mathbf{1} & \mathbf{1} & 0 & \cdots & 0 \\ & & \cdots & & & \\ 0 & \cdots & & 0 & \mathbf{1} & \mathbf{1} \\ \hline \mathbf{1} & 0 & \cdots & & 0 & \mathbf{1} \\ \hline \end{array}}^{p+2}}_{M_{\mathrm{I}_p},\ p \geq 1} \left.\vphantom{\begin{array}{c}1\\1\\1\\1\\1\end{array}}\right\} p+2$$

$$\underbrace{\overbrace{\begin{array}{|cccccc|c|}\hline \mathbf{1} & \mathbf{1} & 0 & & \cdots & & 0 \\ 0 & \mathbf{1} & \mathbf{1} & 0 & \cdots & & 0 \\ & & & \cdots & & & \\ 0 & \cdots & & 0 & \mathbf{1} & \mathbf{1} & 0 \\ \hline 0 & \mathbf{1} & & & \cdots & & \mathbf{1} \\ \mathbf{1} & & \cdots & & \mathbf{1} & 0 & \mathbf{1} \\ \hline \end{array}}^{p+3}}_{M_{\mathrm{II}_p},\ p \geq 1} \left.\vphantom{\begin{array}{c}1\\1\\1\\1\\1\\1\end{array}}\right\} p+3$$

$$\underbrace{\overbrace{\begin{array}{|cccccc|c|}\hline \mathbf{1} & \mathbf{1} & 0 & & \cdots & & 0 \\ 0 & \mathbf{1} & \mathbf{1} & 0 & \cdots & & 0 \\ & & & \cdots & & & \\ 0 & \cdots & & 0 & \mathbf{1} & \mathbf{1} & 0 \\ \hline 0 & \mathbf{1} & \cdots & & \mathbf{1} & 0 & \mathbf{1} \\ \hline \end{array}}^{p+3}}_{M_{\mathrm{III}_p},\ p \geq 1} \left.\vphantom{\begin{array}{c}1\\1\\1\\1\\1\end{array}}\right\} p+2$$

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A (∞, Δ)-matrix can contain

- M_{I_p} with unbounded size,
- M_{II_p} with $1 \leq p \leq \Delta - 2$,
- M_{III_p} with $1 \leq p \leq \Delta - 1$,
- M_{IV}, and M_{V}.

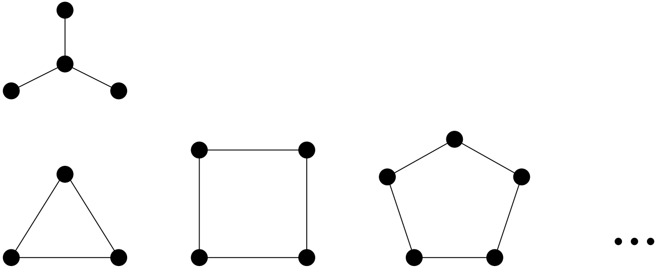
Min-COS-C / Min-COS-R on (∞, Δ)-Matrices

Problem: Matrices M_{I_p} of unbounded size can occur.

Min-COS-C / Min-COS-R on (∞, Δ)-Matrices

Problem: Matrices M_{I_p} of unbounded size can occur.
Idea: Analogy to IDPS.

Forbidden subgraphs for vertex-disjoint unions of paths:



Min-COS-C / Min-COS-R on (∞, Δ)-Matrices

Algorithmic framework for Min-COS-C / Min-COS-R:

1. Destroy the submatrices from

$$X := \{M_{\mathrm{I}_p} \mid 1 \leq p \leq \Delta - 1\} \cup \{M_{\mathrm{II}_p} \mid 1 \leq p \leq \Delta - 2\} \cup \{M_{\mathrm{III}_p} \mid 1 \leq p \leq \Delta - 1\} \cup \{M_{\mathrm{IV}}, M_{\mathrm{V}}\}.$$

2. Destroy the remaining M_{I_p} $(p \geq \Delta)$.

We show:

- We can find a submatrix from X in polynomial time.
- If a (∞, Δ)-matrix M contains none of the matrices in X as a submatrix, then M can be divided into submatrices that have the *"circular ones property"*.
- Min-COS-C / Min-COS-R can be solved in polynomial time on a (∞, Δ)-matrix with the circular ones property.

Min-COS-C / Min-COS-R on (∞, Δ)-Matrices

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We show:

- We can find a submatrix from X in polynomial time.
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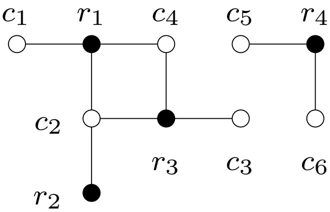
- Min-COS-C / Min-COS-R can be solved in polynomial time on a (∞, Δ)-matrix with the circular ones property.

Min-COS-C / Min-COS-R on (∞, Δ)-Matrices

If a (∞, Δ)-matrix M contains none of the matrices in X as a submatrix, then every component of M has the *circular ones property*.

Components of a matrix:

	c_1	c_2	c_3	c_4	c_5	c_6
r_1	**1**	**1**	0	**1**	0	0
r_2	0	**1**	0	0	0	0
r_3	0	**1**	**1**	**1**	0	0
r_4	0	0	0	0	**1**	**1**



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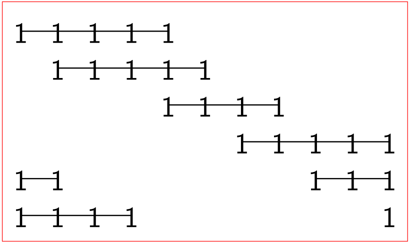


A 0/1-matrix M has the circular ones property if its columns can be permuted such that in each row the ones form a block when M is wrapped around a vertical cylinder.

Min-COS-C / Min-COS-R on (∞, Δ)-Matrices

If a (∞, Δ)-matrix M contains none of the matrices in X as a submatrix, then every component of M has the *circular ones property*.

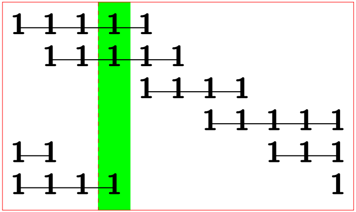
Proof by contraposition:

If a component B of a (∞, Δ)-matrix does not have the circular ones property, then it contains one of the submatrices from X.

Min-COS-C / Min-COS-R on (∞, Δ)-Matrices

Theorem: Let M be a matrix and c_j be a column of M. Form the matrix M' from M by complementing all rows with a 1 in column c_j. Then M has the circular ones property iff M' has the C1P.

[Tucker, Pacific Journal of Mathematics, 1971]



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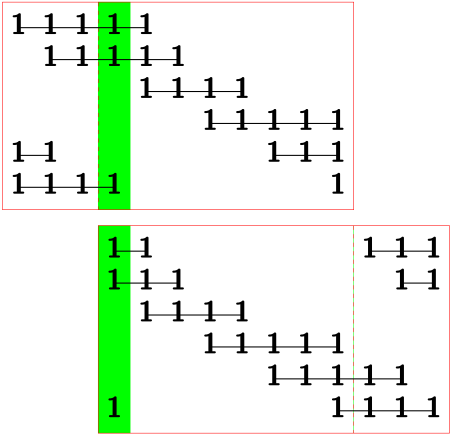

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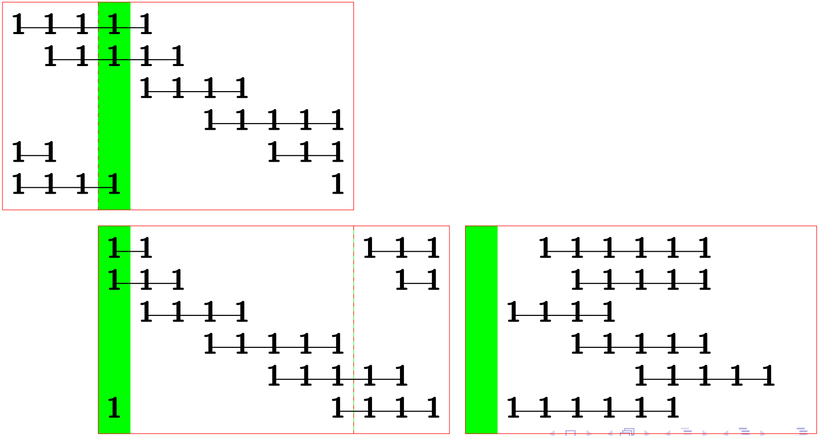
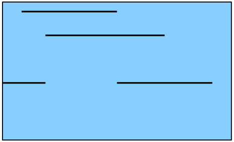


Min-COS-C / Min-COS-R on (∞, Δ)-Matrices

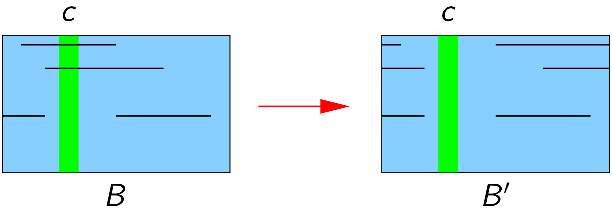
Component B without circular ones property.



B

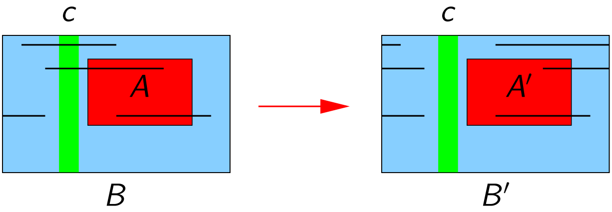
Min-COS-C / Min-COS-R on (∞, Δ)-Matrices

Component B without circular ones property.
$\Rightarrow \exists$ column c such that B' does not have the C1P.



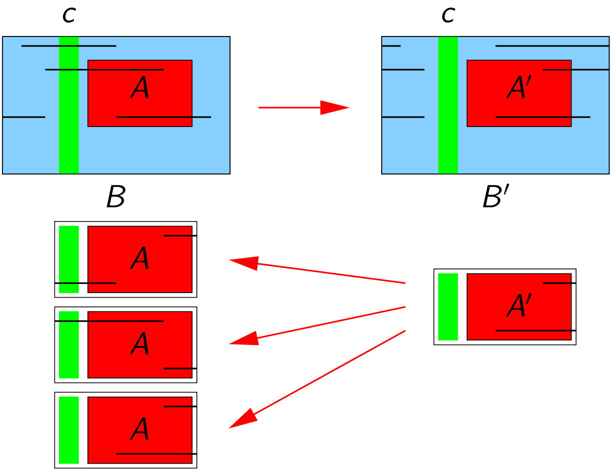
Min-COS-C / Min-COS-R on (∞, Δ)-Matrices

Component B without circular ones property.
$\Rightarrow \exists$ column c such that B' does not have the C1P.
$\Rightarrow$ There is a forbidden submatrix A' in B'.



Min-COS-C / Min-COS-R on (∞, Δ)-Matrices

Component B without circular ones property.
$\Rightarrow$ $\exists$ column c such that B' does not have the C1P.
$\Rightarrow$ There is a forbidden submatrix A' in B'.
$\Rightarrow$ We can always find a submatrix from X in B.



Min-COS-C / Min-COS-R on (∞, Δ)-Matrices

Algorithmic framework for Min-COS-C / Min-COS-R:

1. Destroy the submatrices from

$$X := \{M_{\mathrm{I}_p} \mid 1 \le p \le \Delta - 1\} \cup \{M_{\mathrm{II}_p} \mid 1 \le p \le \Delta - 2\} \\ \cup \{M_{\mathrm{III}_p} \mid 1 \le p \le \Delta - 1\} \cup \{M_{\mathrm{IV}}, M_{\mathrm{V}}\}.$$

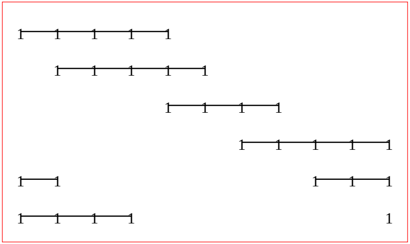
2. Destroy the remaining M_{I_p} $(p \ge \Delta)$.

We show:

- We can find a submatrix from X in polynomial time.
- If a (∞, Δ)-matrix M contains none of the matrices in X as a submatrix, then M can be divided into submatrices that have the *"circular ones property"*.
- **Min-COS-C / Min-COS-R can be solved in polynomial time on a (∞, Δ)-matrix with the circular ones property.**

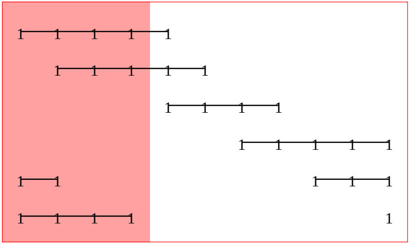
Min-COS-C / Min-COS-R on (∞, Δ)-Matrices

Min-COS-C can be solved in polynomial time on a (∞, Δ)-matrix with the circular ones property.



Min-COS-C / Min-COS-R on (∞, Δ)-Matrices

Min-COS-C can be solved in polynomial time on a (∞, Δ)-matrix with the circular ones property.



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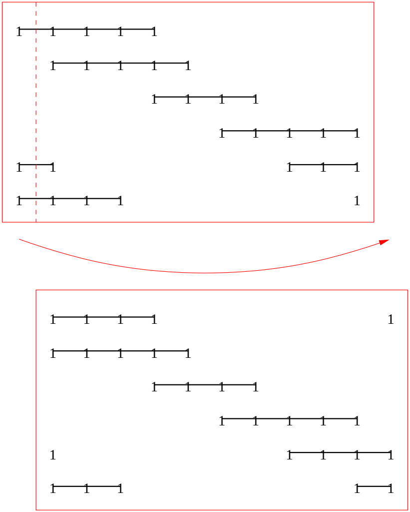

Min-COS-C / Min-COS-R on (∞, Δ)-Matrices

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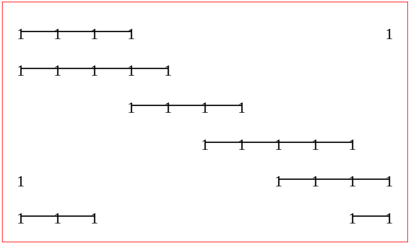

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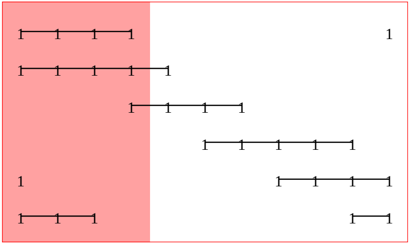
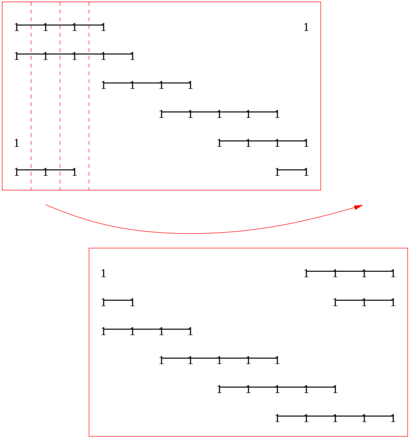
Min-COS-C can be solved in polynomial time on a (∞, Δ)-matrix with the circular ones property.



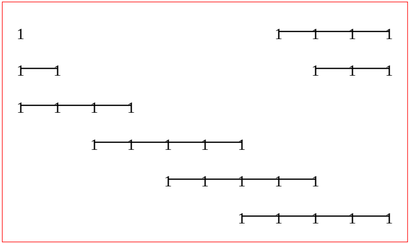
Min-COS-C / Min-COS-R on (∞, Δ)-Matrices

Min-COS-C can be solved in polynomial time on a (∞, Δ)-matrix with the circular ones property.



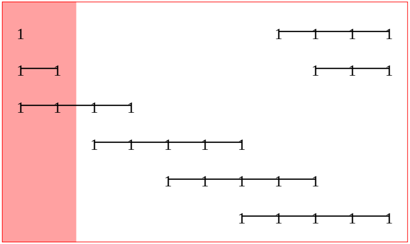
Min-COS-C / Min-COS-R on (∞, Δ)-Matrices

Min-COS-C can be solved in polynomial time on a (∞, Δ)-matrix with the circular ones property.



Min-COS-C / Min-COS-R on (∞, Δ)-Matrices

Min-COS-C can be solved in polynomial time on a (∞, Δ)-matrix with the circular ones property.



Open questions

(1's per col, 1's per row)	Max-COS-C	Min-COS-C
$(3,2)$	0.5-approx[1]	
$(\infty,2)$	• No const. approx.[1] • W[1]-hard	• No 2,72-approx. • Problem kernel
(∞,Δ)	• No const. approx.[1] • W[1]-hard	• $(\Delta+2)$-approx. • $O((\Delta+2)^k \cdot \Delta^{O(\Delta)} \cdot \lvert M\rvert^{O(1)})$-alg.
$(2,3)$	0.8-approx[1]	
$(2,\infty)$	0.5-approx[1]	• 6-approx • $O(6^k \cdot \mathrm{pol}(\lvert M\rvert))$-alg.
(Δ,∞)	?	?

Also open: Deletion of entries instead of columns or rows?

[1] [Tan, Zhang, Algorithmica, 2007]